12) Let V and V' be any two finite dimensional vector space over the same field F. Then prove that the vector space Hom (V, V') of all linear transformation of V to V' is also finite dimensional and dim. Hom (V, V') = dim V × dim V'

13) (i) Prove that two matrices over a field F are similar iff they correspond to the same linear transformation of a vector space V over F to itself with respect to two different bases.

(ii) If t_1 and t_2 are linear transformations of finite dimensional inner product spaces V to V' then prove that :

(a) $(t_1 + t_2)^* = t_1^* + t_2^*$

(b) $(t_1 t_2)^* = t_2^* t_1^*$

Where t^* is adjoint of t.

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MA/MSCMT - 01

December - Examination 2015

M.A./M.Sc. Previous Examination

Advanced Algebra

Paper - MA/MSCMT - 01

Time : 3 Hours] **[Max. Marks :- 80**

Note : The question paper is divided into three Sections A, B, and C. Use of calculator is allowed in this paper.

Section - A 8 x 2 = 16

Note : Section 'A' contain 08 Very Short Answer Type Questions. Examinees have to attempt all questions. Each question is of 02 marks and maximum word limit is thirty words.

1) (i) Define external direct product.

(ii) Write class equation for the finite group.

(iii) Define left module over a ring.

(iv) Define dual space of a vector space V over field F.

(v) Define rank of a linear transformation on vector spaces.

(vi) Define Galois extension of a field.

(vii) Define similar matrices.

(viii)Define self-adjoint linear transformation.

Section - B 4 x 8 = 32

Note : Section 'B' contain 08 Short Answer Type Questions. Examinees will have to answer any four (04) questions. Each question is of 08 marks. Examinees have to delimit each answer in maximum 200 words.

2) Prove that every Euclidean ring is a principal ideal domain.

3) If HK is the internal direct product of H and K then prove that $\frac{HK}{K} \cong H$

4) Prove that the numbers of elements conjugate to 'a' in G is equal to the index of the normalize of a in G.

5) Prove that every additive abelian group is a module over the ring Z of integers.

6) Let A and B be any two matrices of order n × n then prove that: $|AB| = |A|.|B|$

7) If W is any subspace of a finite dimensional inner product space V, the prove that $(W^{\perp})^{\perp} = W$.

8) Let V be a vector space over a field F and B be its basis. Then prove that a linear transformation $t : V \to V$ is invertible iff matrix of t relative to basis B is invertible.

9) If K is a field and $d_1, d_2, ... d_n$ are distinct automorphisms of K. Then prove that it is impossible to find elements $b_1, b_2, ... b_n$ not all zero in K such that $b_1\phi_1(a) + b_2\phi_2(a) + ... + b_n\phi_n(a) = 0 \forall a \in K$

Section - C 2 x 16 = 32

Note : Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions. Each question is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) (i) Prove that every quotient group of a solvable group is solvable.

(ii) Let V be a finite dimensional vector space over a field F. Then Prove that for each non zero vector $v \in V$ then there exists a linear functional $f \in V^*$ such that $f(v) \neq 0$

11) Let K be a Galois extension of a field F. Then there exists a one to one correspondence between the set of all subfields of K containing F and the set of all sub groups of G (K/F). Further if E is any sub field of K which contains F then prove following:

(i) $[K:E] = o[G(K/E)]$ and $[E:F]$ = index of $G(K/E)$ in $G(K/F)$

(ii) E is normal extension of F if and only if $G(K/F)$ is a normal sub group of $G(K/F)$.

(iii) If E is normal extension of F, then $G(E/F) \cong G(K/F)/G(K/E)$.